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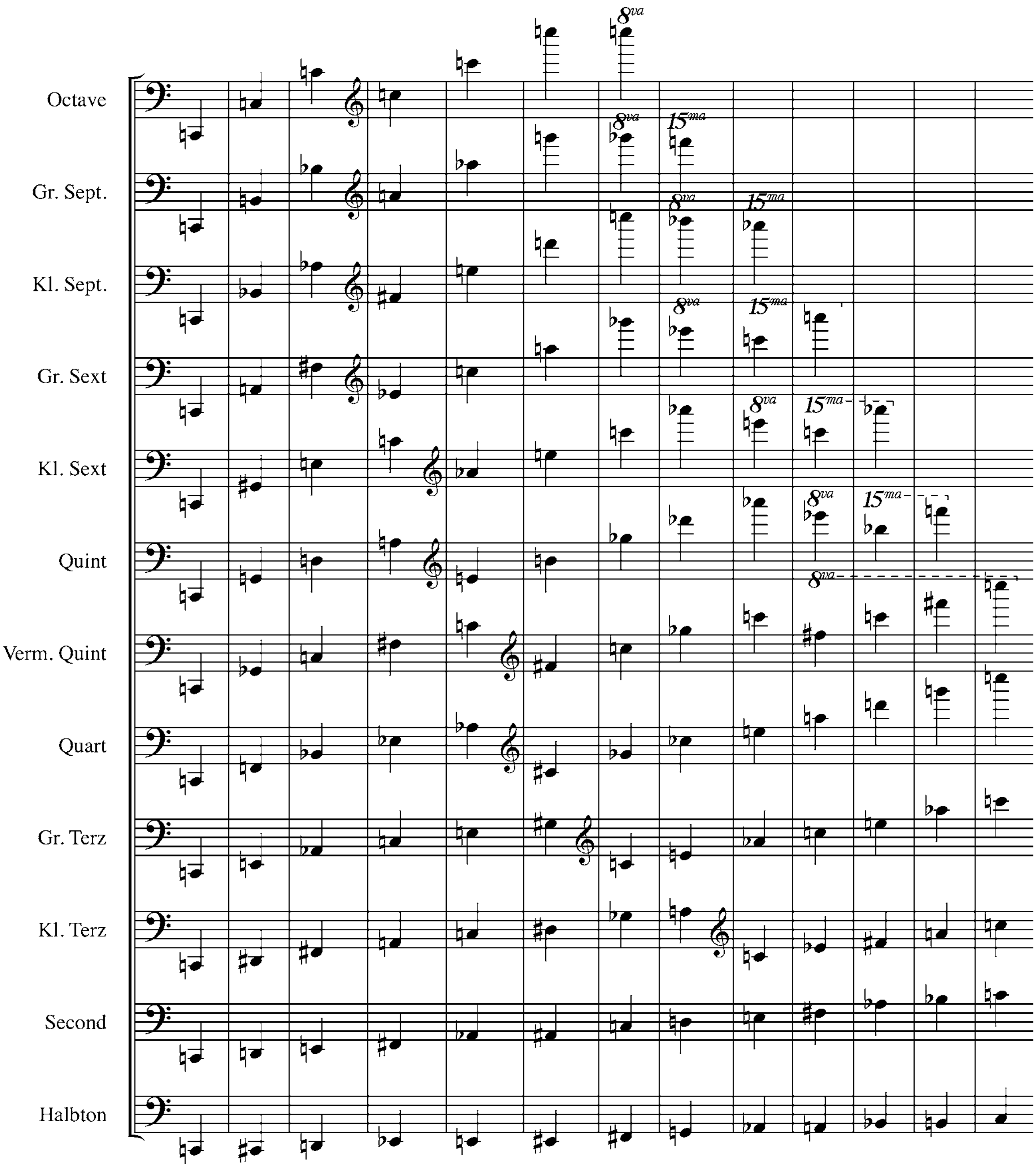
Interval Cycles and Twelve-tone Rows with an Application to the Analysis of Alban Berg's *Lyric Suite*

1. Interval cycles

The role of interval cycles in the music of early 20th century composers is widely recognized. In addition to the analytical observations on the compositions, we have the sketches and the theoretical writings of the composers themselves at our disposal.

The array of interval cycles in Example 1 is particularly interesting. It is an extract from Alban Berg's letter to Arnold Schoenberg, dated July 27th, 1920 (Perle, 1977a). Berg calls this array a "theoretical trifle", but we can see more to it. Dave Headlam (1996) has shown in great detail the

Example 1. Alban Berg's array of interval cycles.



persistence of interval cycles in the music of Alban Berg. Hence, Berg's array of interval cycles, far from being a mere trifle, reflects a significant and continual feature of his musical language, from the second song of his Opus 2 (*Schlafend trägt man mich in mein Heimatland*), which still employs a key signature, through his last work, the twelve-tone opera *Lulu*.

Examples 2 and 3 provide an example of what we call persistence of interval cycles. Example 2 shows the main theme of Alban Berg's *Lyric Suite*. It contains cycles of fifths in a disguised form: taking every second note gives us the cycles *F–C–G*, *E–A–D*, *Ab–Eb–Bb*, and *Db–Gb–B*. The passage in Example 3 is a variant of the main theme, in which the theme is transformed into an explicit cycle of fifths.

Example 2. Main theme of the first movement of Alban Berg's *Lyric Suite* (violin I, measures 2–4).

Example 3. A variant of the main theme of the first movement of the *Lyric Suite* (violoncello and violin I, measures 64–66).

2. Twelve-tone tonality: cyclic set

George Perle (1977b) builds a theory on the interval cycles which he terms *twelve-tone tonality*. We can only scratch the surface of the theory here, picking some elements relevant to our discourse.

Our starting point is the basic concept of Perle's theory: the concept of a *cyclic set*. We begin with an interval cycle – a sequence of pitch classes in which the (ordered) pitch-class interval between adjacent pitch classes is constant. For instance, the two upper rows in Example 4 depict two cycles in which the pitch-class intervals are *7* and *5*, respectively. Both cycles begin with pitch class *0* and the remaining pitch classes are obtained by adding *5* or *7* modulo *12*.[1] (Incidentally, both cycles are from Berg's array.) In order to create a *cyclic set*, we intertwine two interval cycles with complementary pitch-class intervals, such as *5* and *7*. For instance, by intertwining the two interval cycles of Example 4, we obtain the cyclic set that is shown in the bottom line.

An essential feature of cyclic sets is the alternating sums of adjacent pitch-classes modulo *12*. This follows directly from the construction of cyclic sets. For instance, in Example 5 the sums alternate between *0* and *7*.

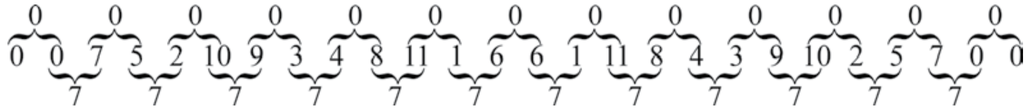
Example 4. A cyclic set obtained by intertwining two interval cycles.

0 7 2 9 4 11 6 1 8 3 10 5 0

0 5 10 3 8 1 6 11 4 9 2 7 0

0 0 7 5 2 10 9 3 4 8 11 1 6 6 1 11 8 4 3 9 10 2 5 7 0 0

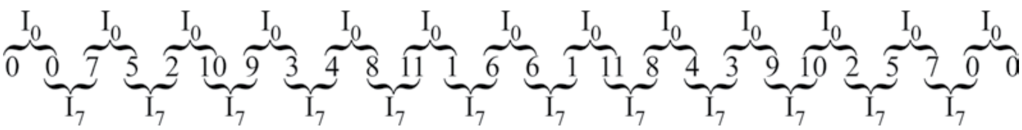
Example 5. Alternating sums (modulo 12) of a cyclic set.



My critique of Perle's presentation of his theory is the concept of sum. Namely, we can argue that pitch classes do not have group structure (or any other algebraic structure for that matter). The notion of "sums of pitch classes", therefore, is ill-advised. Example 5 seems plausible, since it is written with numeric notation. If we were to write it with conventional note names it simply would not make sense. Does *D# + A#* equal *C#*?

Nevertheless, if the "sum" of pitch classes *a* and *b* equals *n*, it means that inversion I_n maps these two pitch classes into each other. Consequently, since the sums of the adjacent dyads of the cyclic set in Example 5 are *0* and *7*, it means that the dyads are invariant under inversions I_0 and I_7, as demonstrated in Example 6. Hence, we can replace the concept of sum with that of inversion. In addition, our aim is to apply the theory to the realm of twelve-tone rows and we are accustomed to discussing twelve-tone theory in terms of inversions, not in terms of sums.

Example 6. Invariant dyads of a cyclic set.

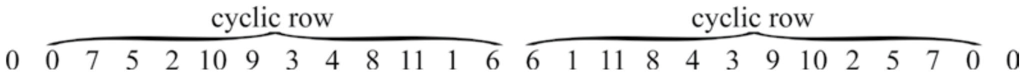


3. Cyclic rows

A cyclic set is not a twelve-tone row and *vice versa*. Nevertheless, cyclic sets and twelve-tone rows are related concepts, since they are both ordered sequences of pitch classes. We can learn something about twelve-tone rows by examining cyclic sets.

There are two major differences between cyclic sets and twelve-tone rows. First, cyclic sets do not necessarily contain all twelve distinct pitch classes. Second, pitch classes may be duplicated in a cyclic set but not in a twelve-tone row. Indeed, the cyclic set in Example 4 begins with a duplication of pitch class *0*. Nevertheless, in certain cases we can extract a segment of twelve distinct pitch classes from a cyclic set and interpret this segment as a twelve-tone row. Example 7 displays two cyclic rows in a cyclic set.

Example 7. Cyclic rows: segments of a cyclic set with 12 distinct pitch classes.



Our preliminary definition of a cyclic row is *a segment of a cyclic set that contains all the twelve distinct pitch classes*. In such cases we can apply the theory of cyclic sets to analyze the properties of cyclic rows.

Many of a cyclic set's properties depend on the cyclic interval. For our purposes, the crucial issue is whether a cyclic set contains all twelve pitch classes. We can formally prove that if the two intertwined interval cycles are ic1-cycles or ic5-cycles, then the cyclic set will always contain all the twelve pitch classes. In addition, if the two intertwined interval cycles are two disjoint ic2-cycles – that is, the two disjoint whole-tone scales – then the cyclic set will also contain all twelve pitch classes. In all other cases, the cyclic set contains less than twelve distinct pitch classes, and such cyclic sets cannot contain any cyclic rows.

We noted earlier that the dyads in Example 5 are invariant under inversions. It so happens that we can define cyclic rows entirely in terms of inversions and we do not need to make any reference to cyclic sets.

Before discussing the details, we should pause for a moment to ask why we should be interested in this phenomenon in the first place. The answer is invariance – a well established means in the art of twelve-tone composition. In short, a composer can bring variety to his piece by employing different musical materials and coherence, through preserving and perhaps even emphasizing invariant features in his materials. For example, in order to add variety, twelve-tone composers usually use several different twelve-tone rows in a composition, and for the purpose of coherence they select rows that are mutually related, often in several different ways.

Let us consider the row of the first movement of Alban Berg's *Lyric Suite*. The row is labeled as *P* in Example 8 and below the row is its inversion I_9P. These rows are thus related by inversion. In addition, if we examine the dyads of the rows – marked with brackets – we notice that the dyads are the same, only reversed. Similarly, in Example 9, we again have row *P* and its inversion I_4P. If we inspect the dyads – again marked with brackets – we notice that the dyads are again the same. This time, five of the dyads are reversed and one is remains unaltered. These rows are thus related both by a transformation and by the dyads.

The invariant dyads are precisely the reason behind our interest in the cyclic rows. Namely, the alternating sums of cyclic sets translate directly into invariance under two inversions in twelve-tone rows. We can take two approaches to the dyads. A stronger approach is to require that all dyads are reversed, such as in Example 8. A weaker one is to require that only the dyads are invariant, as in Example 9.

Example 8. Row *P* of the first movement of Alban Berg's *Lyric Suite* and its inversion I_9P.

Example 9. Row *P* of the first movement of Alban Berg's *Lyric Suite* and its inversion I_4P.

With these preliminary observations we can now give the following definition for cyclic rows in terms of dyads and inversions.

A twelve-tone row is a cyclic row only if the two conditions hold.[2]

(1) The unordered dyads at order positions {0, 1}, {2, 3}, {4, 5}, {6, 7}, {8, 9}, and {10, 11} are invariant under an odd inversion I_n and the unordered dyads at order positions {1, 2}, {3, 4}, {5, 6}, {7, 8}, {9, 10}, and {0, 11} are invariant under inversion I_m.

(2) If *m* is even, then the two pitch classes that inversion I_m keeps fixed must be the first and last pitch classes of the row.

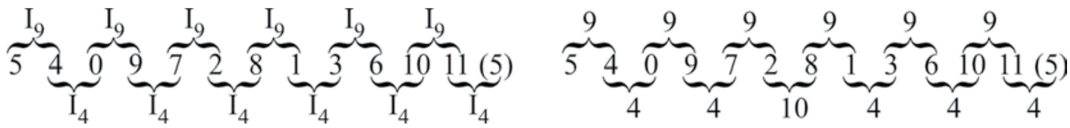
If only the first condition holds, a row is then semi-cyclic.

We can formally show that the preliminary definition of cyclic rows and the aforementioned definition are equivalent, but we will skip the details here.

We defined a stronger and weaker approach to the invariant dyads above. Cyclic rows satisfy the stronger approach to the dyads, semi-cyclic rows satisfy only the weaker approach. We will call rows with shared dyads *cognate rows*.[3]

The *Lyric Suite* row is a semi-cyclic row, but not a cyclic row, because it is not a segment of any cyclic set. This is illustrated in Example 10: even if the dyads are invariant in two inversions, the sums are not the same.

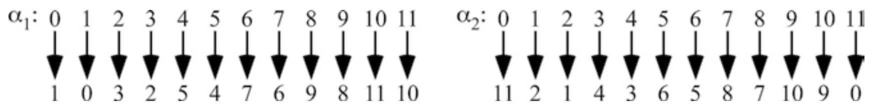
Example 10. *Lyric Suite* row *P* and its dyads and sums.



The pertinence of the phenomenon at hand is reflected in the multitude of approach methods that various authors have proposed. One of these is the family of row operations known as *alpha-operations*, introduced and formalized by Robert Morris (1982). While Morris introduced alpha-operations in terms of pitch classes, we can apply them to the order numbers as well in the spirit of Andrew Mead (1988). As a result of this, we also obtain a compositional practice already described by Ernst Křenek (1960).

Example 11 illustrates alpha-operations as applied to the pitch classes. For example, in operation a_1 we exchange pitch classes *0* and *1*, pitch classes *2* and *3*, etc. Similarly, in operation a_2 we exchange pitch classes *1* and *2*, pitch classes *3* and *4*, etc. Correspondingly, Example 12 illustrates graphically alpha-operations applied to order numbers: we simply exchange adjacent pitch classes. In operation a_1 we exchange order numbers *0* and *1*, order numbers *2* and *3*, etc. Similarly, in operation a_2 we exchange order numbers classes *1* and *2*, order numbers *3* and *4*, etc.

Example 11. Operations α_1 and α_2 applied to pitch classes.



Example 12. Operations α_1 and α_2 applied to order numbers.

If we now return to Example 8, we notice that by applying order number operation a_1 to the upper row *P*, that is, exchanging the adjacent pitch classes of the upper row *P*, we obtain the lower row I_9P. Inverting the row or exchanging adjacent pitch classes, therefore, gives the same result.

Row *P* of the *Lyric Suite* is an exceptional twelve-tone row. Certainly, we do not always obtain an inversion when we exchange adjacent dyads in a row. Sometimes we may get another row form, however. Example 13 shows two rows from the third movement of the *Lyric Suite*, labeled as *Q* and RI_1Q. The row has none of the properties of cyclic rows. Nevertheless, by applying operation a_1 to the order numbers, we obtain retrograde inversion RI_1Q of the original row. In fact, by applying order-number operation a_1 to any row in the row class of row *Q*, we obtain another row in the row class. We call such row classes *alpha-invariant row classes*. The two rows in Example 13 thus provide another example of rows that are mutually related in several different ways: they belong to the same row class (a transformational relation) and they have identical unordered dyads (a shared property).

Example 13. Related row forms in the exposition (first violin, mm. 7–8) and recapitulation (first violin, mm. 45–46) of the first movement of the *Lyric Suite*.

4. Lyric Suite

Let us now apply these concepts to the analysis of the *Lyric Suite*. This is most appropriate, since George Perle uses the row from the first movement to demonstrate issues in his theory, and he cites the piece itself as its originator (Perle 1977a; 1977b).

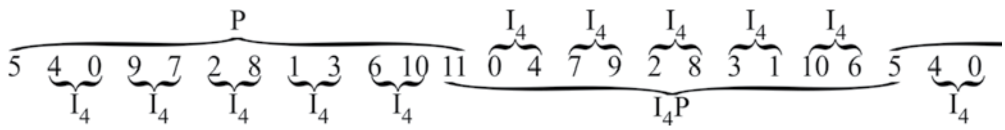
We have already discussed the row forms of the first and third movements that are pertinent to the analysis; Example 14 recapitulates these row forms. Now we need to examine how they are used in the two movements.

Berg uses rows with shared dyads extensively in the first movement of the *Lyric Suite*. Throughout the piece cognate row forms are chained the way row forms *P* and I_4P are chained in Example 15. The first and last pitch classes are shared in consecutive row forms. In addition, the (unordered) dyads in the middle (order numbers *1* to *10*) are always in the same order. In the two row forms the order of each dyad is inverted, except for the middle dyad — in the case of row forms *P* and I_4P these are pitch classes *2* and *8*.

Example 14. Row relations of the first and third movements of the *Lyric Suite*.

P:	5	4	0	9	7	2	8	1	3	6	10	11
I_9P:	4	5	9	0	2	7	1	8	6	3	11	10
P:	5	4	0	9	7	2	8	1	3	6	10	11
I_4P:	11	0	4	7	9	8	2	3	1	10	6	5
Q:	10	9	5	11	0	7	1	6	8	2	3	4
RI_1Q:	9	10	11	5	7	0	6	1	2	8	4	3

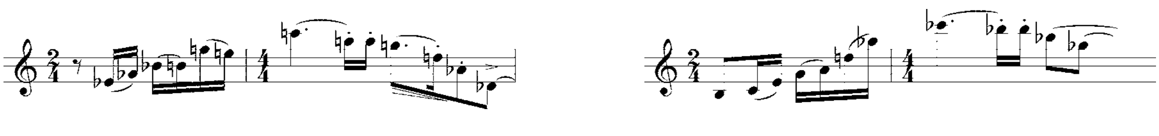
Example 15. Chaining row forms.



As a side note, we point out that the two cognate rows *P* and I_4P (and all corresponding row pairs) are not members of the same hexachord area, since row *P* contains hexachords {*0*, *2*, *4*, *5*, *7*, *9*}/{*6*, *8*, *10*, *11*, *1*, *3*} and row I_4P contains hexachords {*7*, *9*, *11*, *0*, *2*, *4*}/{*1*, *3*, *5*, *6*, *8*, *10*}. Hence, we can consider the cognate relation and hexachord areas as two alternative strategies, both aimed at providing continuity by association with adjacent but distinct row forms.

The first movement of the *Lyric Suite* is in sonata form (Headlam 1996, 250). While the piece is not tonal, some of the features of a tonal sonata form are imitated by the use of the cognate relation. The main theme in the exposition (mm. 2–12) and recapitulation (mm. 42–48) are easily identified. In both cases, the main theme begins with the same row form *P*. In the corresponding measures 7–12 in the exposition and 45–48 in the recapitulation, cognate row forms are used: mainly row form *P* is used in measures 7–12, whereas mainly its cognate counterpart I_4P is used in measures 45–48 (Example 17). Hence, we can interpret the cognate relation as one strategy to bring about coherence in order to compensate for the lack of tonal relations.

Example 16. Cognate rows in measures 7–8 and 45–46 of the first movement of the *Lyric Suite*.



Example 17. Use of rows in the main theme of the first movement of the *Lyric Suite*.

exposition			*recapitulation*		
measure	motive	rows	measure	motive	rows
2–4	*x*	*P*	42–44	*x*	*P*
5–6	*y*	*P*			
7–12	*x'*	*P*	45–48	*x'*	I_4P

As Examples 8 and 9 show, the row of the first movement of the *Lyric Suite* provides two possible dyad relations. It is curious that Berg never uses the perhaps more obvious cognate relation of rows *P* and I_9P – there is not a single appearance of row form I_9P in the first movement. The movement relies on the less obvious relation of row forms *P* and I_4P. For a music analyst this is puzzling: why "waste" such a magnificent row by not taking advantage of its extraordinary properties? (I believe we may safely rule out the possibility that Berg was not aware of this relation.)

The row of the third movement is not a cyclic row, or even a semi-cyclic row. Nevertheless, as Example 14 shows, the row class is invariant under order-number operation a_1. Consequently, row *Q* (that used in the opening of the piece) and row RI_1Q (that used in the closing of the piece) have precisely the same succession of unordered dyads.[4]

In the third movement, we also return with material sharing the dyads, since the row forms used in the recapitulation are retrograded. Hence, we have the retrograded dyads at order positions {0, 1}, {2, 3}, {4, 5}, {6, 7}, {8, 9}, and {10, 11} between row forms *Q* and RI_1Q, as described in Example 14. Thus, in both movements, the recapitulation utilizes a row form that has the same dyads as a row form in the exposition (Example 18).

Example 18. Row relations in first and third movements of the *Lyric Suite*.

	movement I	movement III
form	sonata	*A–B–A'*
dyads	*P* and I_4P	*Q* and RI_1Q

We noted above that of the two invariance relations between dyads, the more obvious one is never utilized in the first movement. We can interpret this as an imbalance that is fixed in the third movement when the relation between dyads at those order positions is used. This suggests a compositional and narrative strategy which could be put in Schoenbergian theoretical terms as a striving for balance from a state of imbalance.

In both the first and third movements the relation of the dyads is used to create long span connections. In the first movement the relation arises from the cyclic origin of the row; in the third movement Berg cleverly emulates the cyclic property with an alpha-invariant row class. Hence, the row relations stemming from the cyclic nature of the rows not only provide local continuity in the *Lyric Suite*, but they also play a structural role both in the first movement and in the third movement.

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Notes

[1] I use a flavor of numeric notation known as fixed-zero notation in this paper: *0* denotes *C*, *1* denotes *C#* or *Db*, etc. In addition, I use numeric notation for the order numbers (Babbitt 1960).

[2] I adopt Andrew Mead's convention of writing order numbers and order-number operations in bold (Mead 1988).

[3] Compare to Perle (1977, 22).

[4] Incidentally, this invariance feature would allow dividing both rows into intertwined (ordered) hexachords *A50183* and *9B7624,* which is a rare property; this device is not used in this movement, however.

Santrauka

Intervalinių ciklų ir serijų taikymas analizuojant Albano Bergo „Lyrinę siuitą“

G. Perle'io dvylikatonės tonacijos teorija, kurios pagrindas yra ciklai, randami A. Bergo ir kitų XX a. kompozitorių muzikoje, pagal priimtą šių terminų vartojimo prasmę nėra nei dvylikatonė, nei tonali. Nepaisant to, kiek pakoreguoti, šios teorijos elementai yra aktualūs, tyrinėjant serijų šeimas. Perle'io teorijos esmė yra ciklinių setų – dviejų susipynusių intervalinių ciklų su papildomais garso aukščio klasės intervalais sekos – koncepcija. Pvz., cikliniame sete 012B496785A3 yra du susipynę pilnų tonų garsaeiliai. Arba, kitaip sakant, ciklinį setą charakterizuoja dvi kintamos sumos. Pastarajame pavyzdyje kintamos sumos yra 1 ir 3.

Šiame pranešime pirmiausia apibūdinamos ciklinių ir pusiau ciklinių serijų sąvokos, o po to apibrėžiami šiuo būdu susiję serijų šeimų setai, turintys kompoziciškai sugestyvių savybių. Vietoj Perle'io sumų ir skirtumų mes pritaikome šią teoriją serijoms, transpoziciškai ir inversiškai pakeisdami atitinkamas struktūras, susiejame šias struktūras su serijų formų, dar vadinamų *alfa* formomis, kurias apibrėžė ir tyrinėjo R. Morrisas, šeima. Analitiškai taikant naujai interpretuotą Perle'io teoriją, nagrinėjamos serijų struktūros pirmoje ir trečioje A. Bergo „Lyrinės siuitos“ dalyse. Aptarus ciklinę serijų kilmę, jos siejamos pagal serijų šeimas, parodant ryšius tarp serijų formų, kurios suteikia ir vietinės reikšmės tęstinumo, ir, veikdamos kaip jungtys, sustiprina ilgalaikius ryšius tarp kūrinio dalių. Šis Perle'io teorijų pritaikymas Bergo muzikai yra tartum ciklo pabaiga, nes būtent „Lyrinė siuita“ ketvirtojo dešimtmečio pabaigoje davė Perle'iui akstiną pradėti plėtoti jo teoriją.