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On verticals in Stravinsky's "Abraham and Isaac"

1. "Abraham and Isaac" is a sacred ballad for baritone and small orchestra composed by Igor Stravinsky in 1962-63 on the Hebrew text of Genesis chapter 22, verses 1-19. In my opinion this work is one of the three most important compositions in the 20th century written on Hebrew text; the two others are "Psalm 130" by Arnold Schoenberg and "Tehillim" by Steve Reich.

2. The work is serial in design, but rather than the full 12-tone series, its two hexachordal units are stressed:

a	b
7 8 10 0 1 9	11 3 2 4 6 5

3. The pitch material of the piece has been derived from this series subjecting the hexachords of its P, R, I, RI, IR and I(RI) forms to the operation of rotation-transposition (Ex. 1).

4. The horizontal lines of the piece has been derived almost without exception by means of horizontal reading of the hexachordal arrays shown in Ex. 1.

5. The verticals of the piece can be divided into two classes:
a) the verticals as intersections of the contrapuntal lines;
b) the verticals as stressed chords.

6. In the following I shall speak about the verticals as stressed chords and, more exactly, about the subspecies of the latter, about the so called "serial verticals".

7. It is generally accepted among theorists that the practical use of such kind of verticals by Stravinsky is of great importance for the development of the 12-tone serialism in the second half of the 20th century.

8. Serial verticals are derived directly by means of vertical reading of some hexachordal rotational array. As the hexachordal rotational arrays are duplicating (i.e. there is pitch-class duplication in columns other than first) "serial" octaves and unisons occur.

9. In the rest part of my paper I would like to introduce to you the technique by means of which Stravinsky employs the serial verticals in "Abraham and Isaac". I am not going to interpret, why did Stravinsky choose exactly these verticals and not others. This kind of interpreting (in fact, a very exciting one) would be a proper subject for another paper.

10. I would call the serial verticals "intentional" (Ex. 2), for they can be subjected to the proved serial system of the work, contrary to the contrapuntal vertical, about the intentionality of which I have only hypotheses.

11. Each hexachordal vertical in the table of Ex. 1 corresponds to one pitch class set. You can find them on the left side of Ex. 2. The number of the sets is limited, namely 6. I would call them "normal serial verticals" (Npcs).

12. However, for the sake of greater variety Stravinsky does not confine himself only to the 6 normal serial verticals, but also transforms them in several ways (on the right side of Ex. 2 I have listed 22 transformational sets (Tpcs).

a) Vocal mixing (vmix.). Stravinsky uses normal verticals in the instrumental parts and "mixes" them by means of the notes of the vocal part.

b) Insertion (ins.). Stravinsky inserts the first unisono-vertical of the hexachordal array to some other vertical of the same array.
c) Addition (add.). Stravinsky adds to normal vertical some extra pitches.
d) Substitution (sub.). Stravinsky substitutes some pitch or pitches of the normal vertical with some other pitch or pitches.
e) Elimination (elim.). Stravinsky eliminates some pitch or pitches from normal vertical.
f) Selection. Stravinsky selects some pitches from normal vertical.

13. In "Abraham and Isaac" I have found 9 cases of serial verticals.
1) The serial verticals of Pb, mms 69-72 (Ex. 3). Here Stravinsky uses insertion, vocal mixing and addition (the pitch class numbers of the transformational notes are always in the brackets).
2) The serial verticals of IRb, mms 87-90 (Ex. 4). Here Stravinsky uses vocal mixing and substitution. In this example the first, unisono-column of the hexachordal array is missing.
3) The serial verticals of Pb, mms 173-177 (Ex. 5). Here the first column is also missing and only vocal mixing is used.
4) The serial verticals of IRa, mms 182-183 (Ex. 6). Here substitution and addition are used. As the section is instrumental, vocal mixing is missing.
5) The serial verticals of IRb, mms 195-196 (Ex. 7). This is the only case out of the nine, where only normal verticals are used.
6) The serial verticals of IRb, mms 203-206 (Ex. 8). This is the instance of elimination. Vocal mixing is also used.
7) The serial verticals of IRb, mms 220-222 (Ex. 9). Vocal mixing and addition is used.
8) The serial verticals of Rb, mms 229-239 (Ex. 10). This example includes another instance of insertion, this time by vocal mixing.
9) The ninth case is hypothetical as its intentionality is only asserted by me. I cannot prove it. Here the music is constructed as succession of dyads. I suppose that here Stravinsky has used, horizontally and vertically, the partial verticals of the array of IRa, but I have not found any mathematical algorithm for this procedure yet.

14. Summing up, one can say that Stravinsky, who has called himself "a vertically thinking composer", has in "Abraham and Isaac" (and also in other works of 1960ies) worked out a serially based method in order to gain a variety of verticals full of "serial" octaves and unisons, the verticals able to suit his sophisticated needs. But the question still remains, why did Stravinsky chose exactly these verticals and not others. Perhaps the comparative analysis of the verticals of the whole production of Stravinsky gives us the answer.

Ex.1

	P		R	
	a	b	a	b
0	7 8 10 0 1 9	11 3 2 4 6 5	5 6 4 2 3 11	9 1 0 10 8 7
1	7 9 11 0 8 6	11 10 0 2 1 7	5 3 1 2 10 4	9 8 6 4 3 5
2	7 9 10 6 4 5	11 1 3 2 8 0	5 3 4 0 6 7	9 7 5 4 6 10
3	7 8 4 2 3 5	11 1 0 6 10 9	5 6 2 8 9 7	9 7 6 8 0 11
4	7 3 1 2 4 6	11 10 4 8 7 9	5 1 7 8 6 4	9 8 10 2 1 11
5	7 5 6 8 10 11	11 5 9 8 10 0	5 11 0 10 8 9	9 11 3 2 0 10

	I		IR	
0	7 6 4 2 1 5	3 11 0 10 8 9	5 4 6 8 7 11	1 9 10 0 2 3
1	7 5 3 2 6 8	3 4 2 0 1 7	5 7 9 8 0 6	1 2 4 6 7 5
2	7 5 4 8 10 9	3 1 11 0 6 2	5 7 6 10 4 3	1 3 5 6 4 0
3	7 6 10 0 11 9	3 1 2 8 4 5	5 4 8 2 1 3	1 3 4 2 10 11
4	7 11 1 0 10 8	3 4 10 6 7 5	5 9 3 2 4 6	1 2 0 8 9 11
5	7 9 8 6 4 3	3 9 5 6 4 2	5 11 10 0 2 1	1 11 7 8 10 0

	I(RI)
0	9 10 8 6 7 3 1 5 4 2 0 11
1	9 7 5 6 2 8 1 0 10 8 7 9
2	9 7 8 4 10 11 1 11 9 8 10 2
3	9 10 6 0 1 11 1 11 10 0 4 3
4	9 5 11 0 10 8 1 0 2 6 5 3
5	9 3 4 2 0 1 1 3 7 6 4 2

	RI
0	9 8 10 0 11 3 5 1 2 4 6 7
1	9 11 1 0 4 10 5 6 8 10 11 9
2	9 11 10 2 8 7 5 7 9 10 8 4
3	9 8 0 6 5 7 5 7 8 6 2 3
4	9 1 7 6 8 10 5 6 4 0 1 3
5	9 3 2 4 6 5 5 3 11 0 2 4

Ex.2 "Intentional" verticals in "Abraham and Isaac"

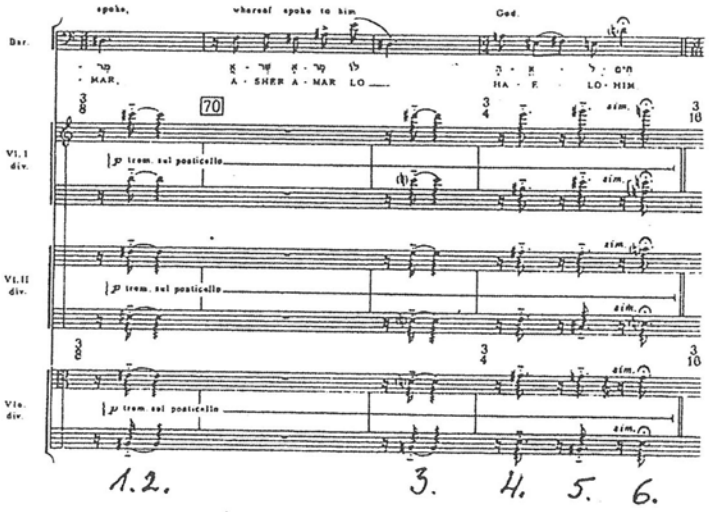
The pitch class sets of the...

normal serial verticals...		and their transformations...			
4-Z15	5-29	3-7	4-5	5-8	6-Z12
4-21	5-Z36	3-9	4-16	5-9	6-Z23
4-22			4-27	5-16	6-Z24
4-25			4-Z29	5-24	6-Z27
			4-13	5-25	6-33
				5-28	6-Z45
				5-31	
				5-33	
				5-35	

...by means of

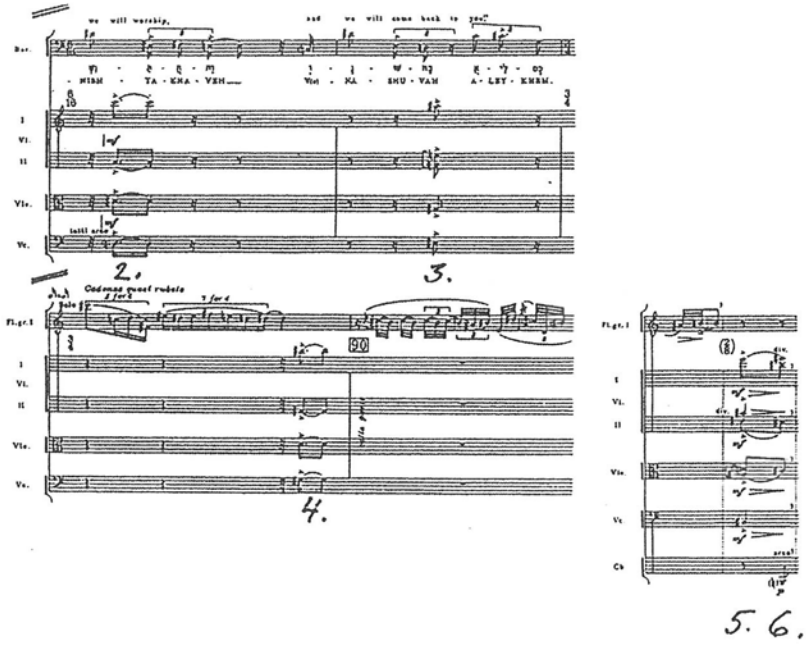
- vocal mixing
- insertion
- addition
- substitution
- elimination
- selection.

Ex.3 (mms 69-72: ins., vmix., add.)



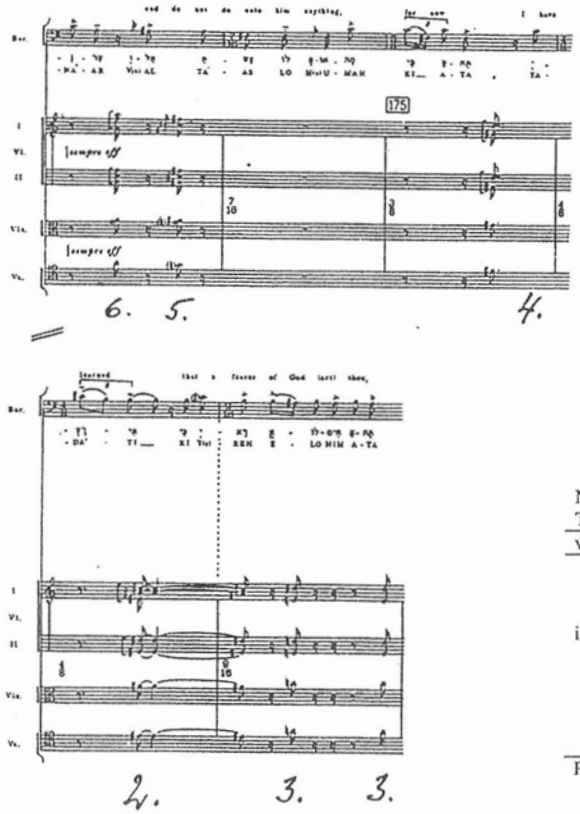
Npcs		4-22	5-Z36	4-21	4-21	5-Z36	5-Z36	4-22
Tpcs		5-24		5-24	5-8		6-Z45	6-33
voc.		3	3	(9)←vmix.→(5)		6	(4)←vmix.→(2)	
		1	3	4	4	8	8	7
	11→ ins.→(11)		0	8	8	1	1	0
		3	4	6	6	6	6	5
instr.		1	0	2	2	10	10	9
		5	2	8	8	7	7	0
		10	9	2	2	10	10	5
								add.→(2)
Pb	1.	2.	3.	4.	4.	5.	5.	6.

Ex.4 (mms 87-90: sub., vmix.)



Npcs		4-Z15	4-Z15	4-Z15	5-29	5-29	5-29	4-25	5-29	4-Z15
Tpcs			5-25	5-31	6-Z23	6-Z24	6-27	5-28	5-9	4-5
voc.		3	(5) ←	(6)←vmix.→(2)		→(0)	→(1)	→(3)		
	11sub.→(0)	→→(0)	→→(0)		10	10	10	0	4	5
		9	9	9	4	4	4	6	10	0
instr.		3	3	3	7	7	7	2	7or9sub.→(3)	3sub.→(1)
		2	2	2	0sub.→(8)	→→(8)	→→(8)	8	2	→(1)
					5	5	5		10	0
									7or9sub.→(0)	11
IRb		2.	2.	2.	3.	3.	3.	4.	5.	6.

Ex.5 (mms 173-177: vmix.)

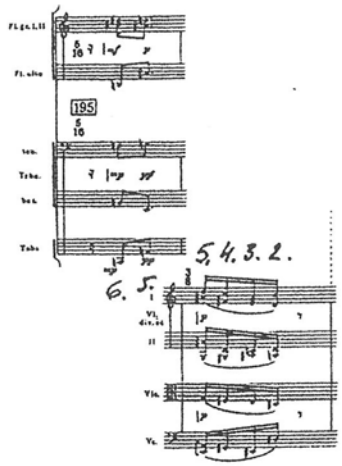


Npcs Tpcs	4-22	5-Z36	4-21	4-21 5-33	4-22 5-24	5-Z36 6-Z12	5-Z36
voc.	9	10	6	(0)←vmix.→(11)←vmix.→(10)			9
instr.	9	6	4	4	1	3	3
	5	1	2	2	10	4	4
	9	8	8	8	5	2	2
	5	6	4	4	3	4	4
	0	1	2	2	5	0	0
	7	8	6	6	10	9	9
	9	10	8	8	5		
	7	7	6	6			
	9	7					
Pb	6.	5.	4.	4.	2.	3.	3.

Ex.6 (mms 182-183: sub.. add.)

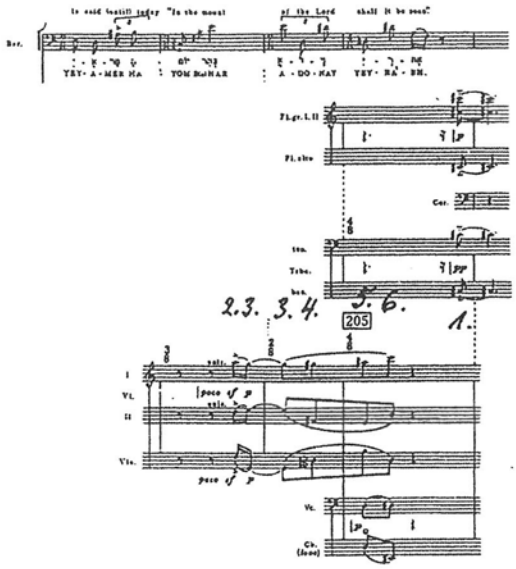
Npcs Tpcs		4-22	5-Z36 5-16	4-21 6-33	5-Z36	4-22
instr.	5	7	6	10	4	6
	5	4	9	8	0	3
	5	4	6	2	7	11
	5	11	8sub.→(7)	add.→(7)	1	1
	5	9	10	add.→(5)	2	3
			3	0		
IRa	1.	2.	3.	4.	5.	6.

Ex.7 (mms 195-196: Npcs only)



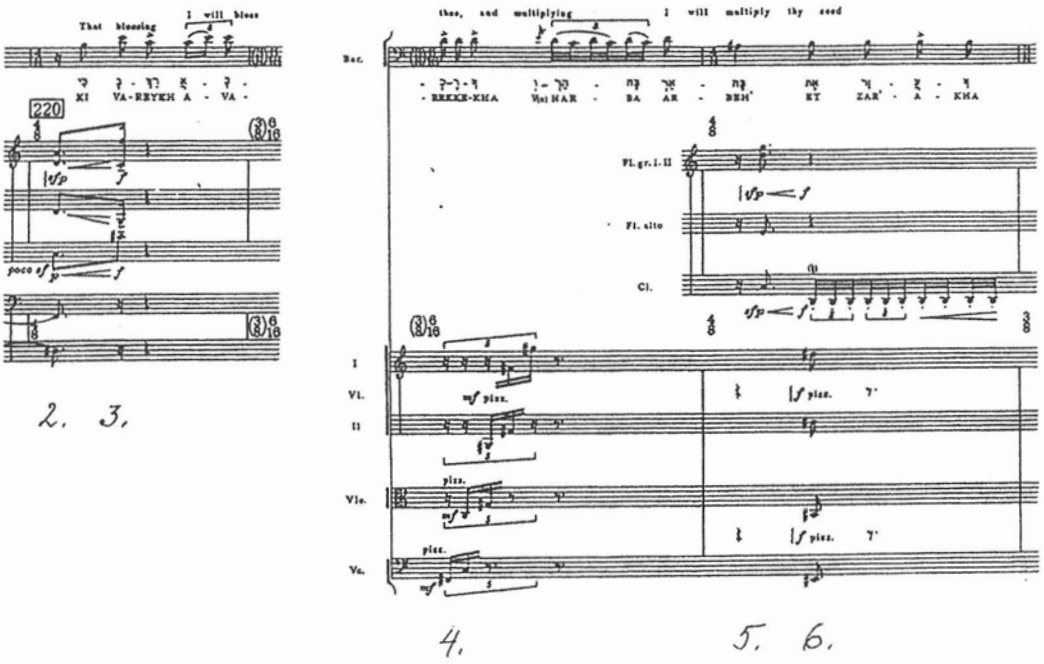
Npcs	4-Z15	5-29	5-29	4-25	5-29	4-Z15
Tpcs						
	3	2	2	0	7	11
	0	10	10	6	4	2
	11	10	10	2	0	2
instr.	5	7	7	8	10	9
	11	4	7	6	4	3
	0	9	9	8	5	3
			4			
IRb	6.	5.	5.	4.	3.	2.

Ex.8 (mms 203-206: elim., vmix.)



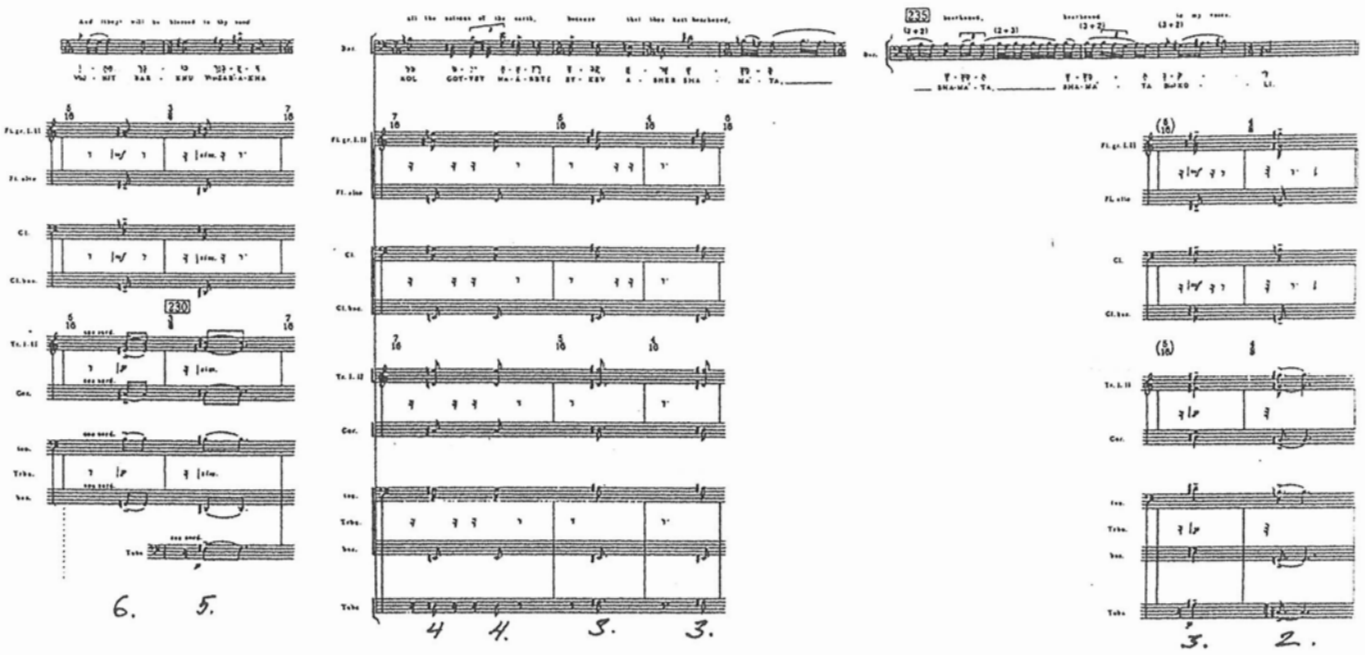
Npcs	4-Z15	5-29	5-29	4-25	4-25	5-29	4-Z15	
Tpcs	3-9		3-7		4-27	4-16		
voc.	(4)←vmix.	4	(2)←vmix.		→(2)←vmix.→(3)	2	3	
	9	7	7	8	8	10	0	1
	2	4	4	6	6	4	11	1
instr.	2	4	4	0	0	9	0	1
	elim.→3	elim.→0	elim.→0	elim.→2	elim.→2	2	3	1
	elim.→3	elim.→5	elim.→5			2	5	1
	elim.→11	elim.→10	elim.→10			elim.→7		
IRb	2.	3.	3.	4.	4.	5.	6.	1.

Ex.9 (mms 220-222: vmix., add.)



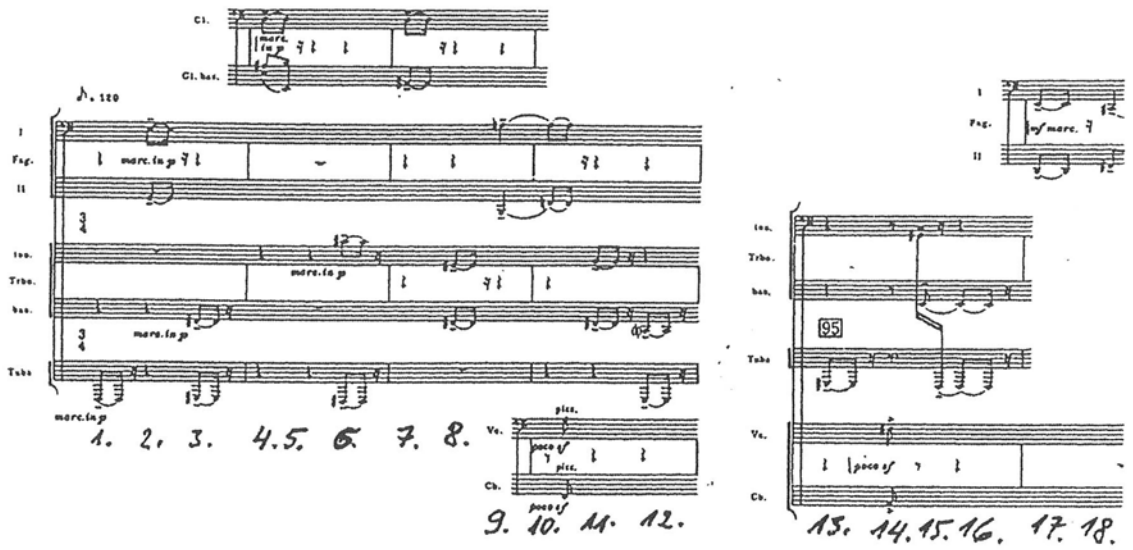
Npcs	4-Z15	4-Z15	5-29	4-25	5-29	4-Z15
Tpcs	4-Z29	5-28				
voc.		(11)←vmix.	4	2	10	11
	9	9	5	8	7	11
	2	2	0	6	2	3
	2	2	7	6	4	0
instr.	9	9	10	8	9	0
	(5)← add. →(5)			8		5
	3	3		0		
				0		
				6		
IRb	2.	2.	3.	4.	5.	6.

Ex.10 (mms 229-239: vmix., ins.)



Npcs		4-Z15	4-Z15	5-29	5-29	5-29	5-29	4-25	4-25	4-25	5-29	5-29	5-29	5-29	5-29	5-29	4-Z15
Tpcs		5-9			6-33			5-28						6-33			
voc.	9ins.→(9)		7	8vmix.→(10)		0	1 vmix.→(11)		2	4	6	4	3 vmix.→(8)		10	0	1
		11	11	1	1	1	1	4	4	4	6		6	6	6		8
		10	10	8	8	8	8	10	10	10	0		0	0	0		8
		5	5	0	0	0	0	2	2	2	3		3	3	3		7
		11	11	6	6	6	6	8	8	8	10		10	10	10		11
instr.		7	7	3	3	3	3	4	4	4	5		5	5	5		7
		11	11	1	1	1	1	4	4	4	6	6	6	6	6	6	8
		5	5	8	8	8	8	2	2	2	3	3	3	6	6	6	7
		10	10	0	0	0	0	10	10	10	0	0	0	0	0	0	8
		11	11	0	0	0	0	8	8	8	10	10	10	3	3	3	11
		7	7	3	3	3	3	4	4	4	5	5	5	5	5	5	7
				6	6	6	6	2	2	2	6	6	6	10	10	10	1
Rb	1.	6.	6.	5.	5.	5.	5.	4.	4.	4.	3.	3.	3.	3.	3.	3.	2.

Ex.11 (mms 91-96: selection)



IRa

1.	2.	3.	4.	5.	6.				14.	13.	12.	11.
5	4	6	8	7	11	16.	5	4	6	8	7	11
5	7	9	8	0	6		5	7	9	8	0	6
5	7	6	10	4	3		5	7	6	10	4	3
5	4	8	2	1	3	15.	5	4	8	2	1	3
5	9	3	2	4	6	10.	5	9	3	2	4	6
5	11	10	0	2	1		5	11	10	0	2	1
		8.	7.					9.			17.	18.