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Several Mathematical Aspects of Music Notation in the 20th Century Music Composition

This article continues the musicological research about music and mathematics connection problem. Parallel between music and mathematics looks back to the period of Antiquity. Then the music was a branch of science *quadrivium* – the music was alongside with and equivalent to other exact sciences, such as arithmetic, geometry and astronomy. Mathematical point of music manifests in later periods also. In the 20th century music composition we may see the synthesis of various numerological traditions of earlier epochs.

I would analyse one of the aspects of music and mathematics interaction. It is connected with a question of music notation. Generally, the use of mathematical phenomenon in music composition may be analysed in various ways – from investigation of composing techniques with mathematical background to symbolic incrustations of mathematical aspect into music material. For example, it is well known the case to signify the music tones with the numbers in twelve-tone and serial music practice – the transcription of music pitches, duration or dynamics into digits, the transform of chromatic tone scale into number sequence from 1 to 12.

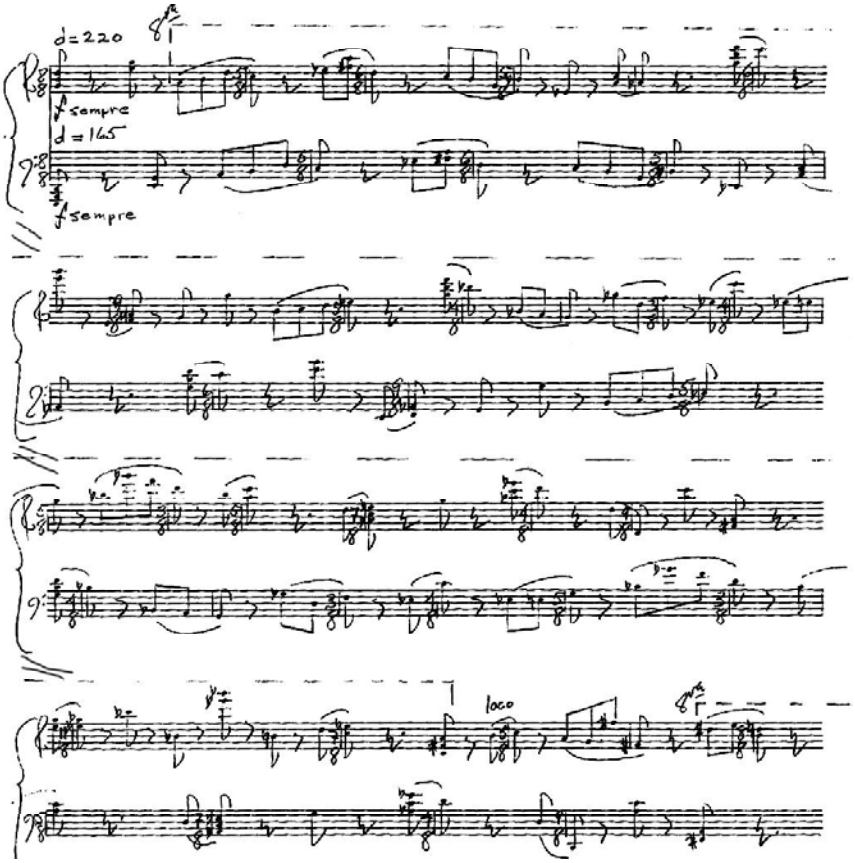
I would address to the examples of 20th century music compositions where the notation may be interpreted mathematically. But at first I would like to concretize the use of the term "mathematical" music notation. A term was invoked in general view and I appeal to the definition of mathematics – as a research of abstract structures, using logics and mathematical markings.

The mathematical aspects of music notation I would differentiate: the first, the use of abstract or pure mathematical elements in the notation procedure. I would call it a technical aspect of music composition. The second, also a technical aspect is a particular mathematical construction, which becomes the initial idea or prototype of musical composition, the possibility to transform the traditionally notated music composition into constructive-mathematical model. And the third concerns the semantic aspect – it means that the mathematical model or the mathematically notated music composition manifests as a semantic code.

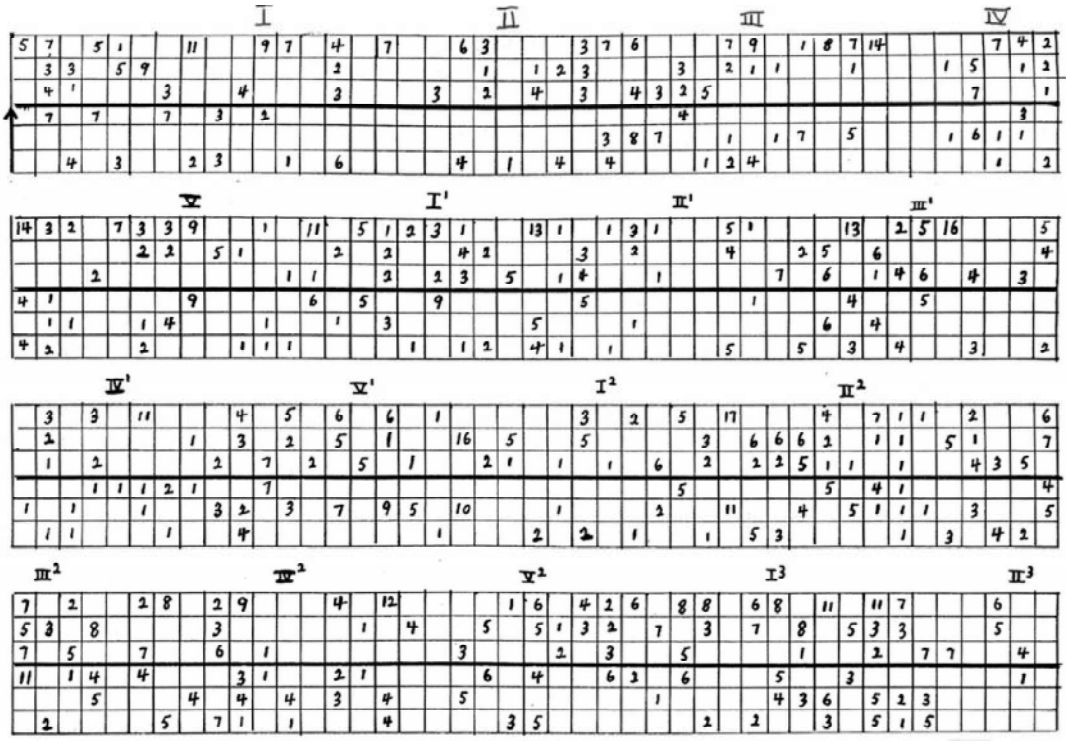
The first aspect includes the use of direct mathematical elements for music composition notation. For example, one of elements of notation, the tempo parameter may become an example of complex mathematical expression. The Studies for piano playing by Conlon Nancarrow represent the original composer's attempt to coordinate the tempos of separate voices of the score with the use of mathematical proportions. For example, Study N. 15 is an exact canon for 2 voices. The balance of two different tempos correspond the formula ¾ and the title of the Study is marked "Canon ¾" (Example 1). Next two examples of Studies overpass the trivial poly-metric boundaries and become transcendental music compositions. For example, in the beginning of the Study N. 27 in 4 voices we read the title by composer "Canon 5%-6%-8%-11%". This marking of percents means the proportion in which the tempo or the movement of each of 4 voices is harmonized. Another example is the Study N. 37 for 12 voices: here the tempo marking of every voice composes into the digital row 150-160$^{5}/_{7}$-168¾-180-187$^{1}/_{2}$-200-210-225-240-250-262$^{1}/_{2}$-281$^{1}/_{4}$. The 12 different tempos inter-coordinate according to these number proportions. And this digital row is not accidental – it is known that these mathematical proportions express the relations between the successive tones of all the chromatic scale and Nancarrow was influenced in such complex mathematical expressions.

These examples concern the particular parameter of music composition. Also the whole music score may be digital apparently. It is Morton Feldman's score "IXION" that is full of the digits in the panes on the paper only (Example 2). Composer used the numbers as symbols for notation and the numbers determined the amount of tones. Already in the first composition "Extensions 1" (1951) that was notated in the same way – it means the numbers were written on the special graphic paper – the numbers signified the amount of played notes, the lines of paper signified the register, but the concrete pitch of tones was chosen by the interpreter. The analogous numerical symbols of music notation are used in the mentioned "IXION" as in the score of the second, third of fourth "Intersections". This "IXION" score version is written for the ensemble of two pianos. The central black line indicates the partition of the pianos. According to the explanation how to perform this

Example 1. Conlon Nancarrow's Study N. 15 "Canon ¾". The formula ¾ is represented in the tempos marking 165 : 220 = 3 : 4



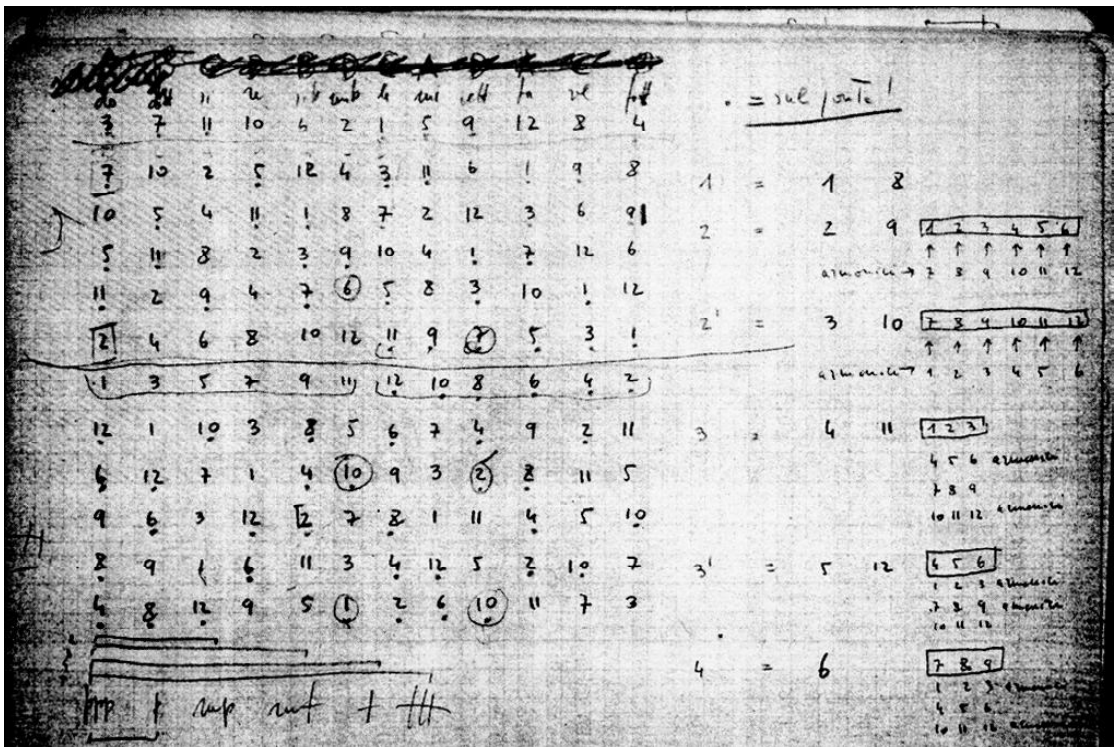
Example 2. Morton Feldman's "IXION" – the "digitised" score for two pianos



composition the vertical of three panes means the approximate partition into three registers – the high, middle and the low. One pane is equal to the duration of one second and the number in the pane means the amount of tones. The interpreter decides what pitches to play and in what manner – to play all tones together (harmonic) or in course (melodic). Prima facie the score of "IXION" seems a chaotic dispersion of numbers and the composer eliminated whatever constructive moment because he tried to create a phenomenon of a chance. But after detailed analysis it was observed that composer uses even the isomorphic principle – in the middle of composition Feldman exactly repeats two sections one after another.

The second aspect may be illustrated by well known example of Webern and number square, which composer used in his serial composition. Also it is known a sketch of number square that was used by Luigi Nono. Composer adjusted the sequence of the square to the rhythmical parameter in his piece "Varianti" (Example 3). And so we may say that such a mathematical sketch, also a model, pattern or constructive frame becomes a prototype for music composition before its notation. I would call it a primary tool for the further composing process.

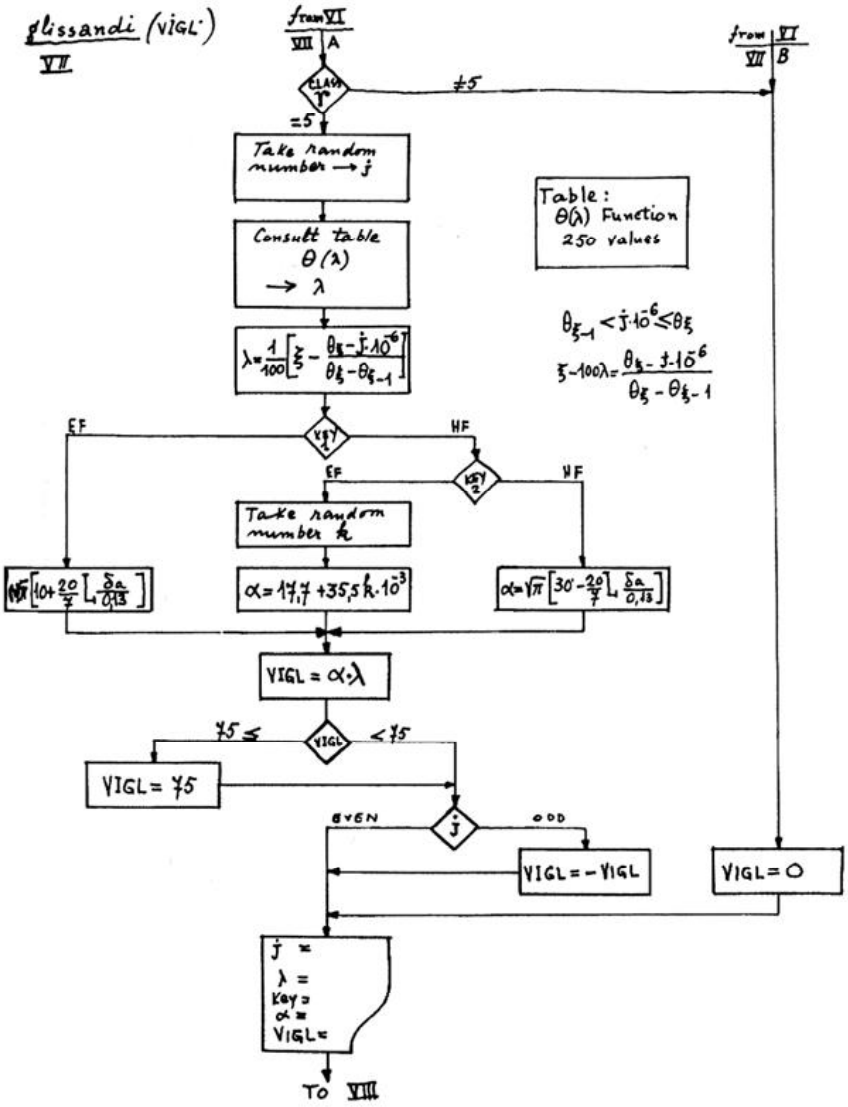
Example 3. Luigi Nono's "Varianti". The number square sketch – a model for rhythmical organization



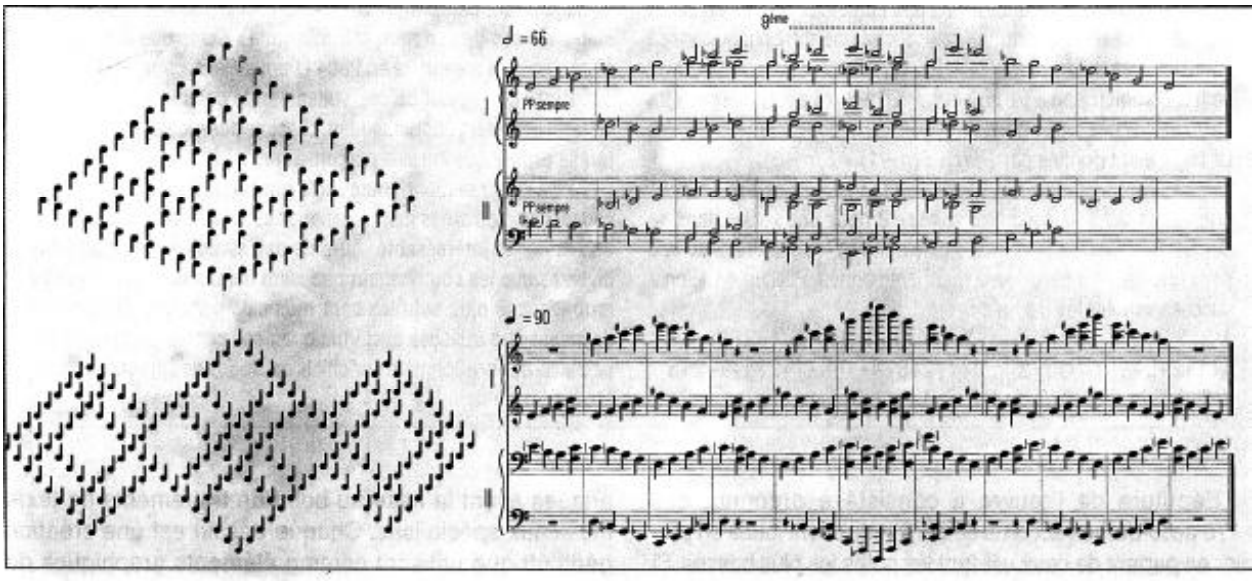
Another example shows that the primary idea, the primary model of music composition is a complex mathematical structure. It is a sketch by Xenakis for his composition "Achorripsis" (Example 4).

The graphical algorithm also may become a primary mathematical model of music composition. The music composition may be notated in traditional way after the creation of the graphical algorithm. The graphics may influence and determine the concrete parameter of music. For example, Lithuanian composer Šarūnas Nakas in his composition "Ziqquratu" initially used graphical charts for piano clusters. According to composer's remarks the graphic charts were transformed into piano part. After analysis and detailed recording the cluster scales it was observed the symmetry between the curved forms. Composer divided the symmetrical form into parts and incorporated them in the various sections of the composition. Another example of symmetrical graphical expression of musical notation may be illustrated by Tom Johnson's composition for piano in four hands "Symmetries" (1981–1990). The cycle consists of 49 little pieces and you may see in the example a symmetrical structure made by composer and its application – musical harmonization (Example 5).

Example 4. Xenaki's "Achorripsis". The sketch of mathematical formulas – a prototype for the composition

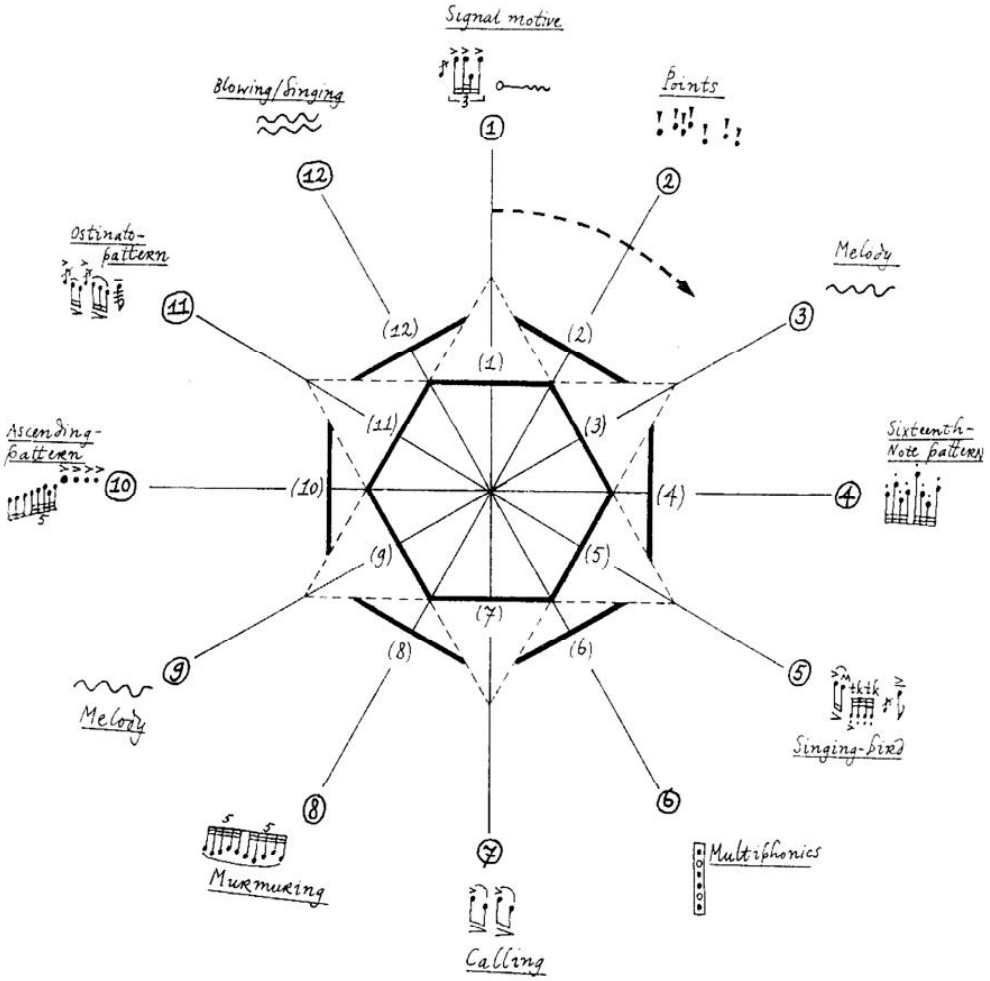


Example 5. Tom Johnson's "Symmetries". A symmetrical pattern involved in the piano pieces



But the graphical example may become the model of the whole music composition. In the beginning of Jan Rokus Roosendael's composition "Rotations" score you may see a hexagonal figure and various possibilities of its rotations and combinations of the corners. This hexagonal figure was used for the composing process of the whole music piece (Example 6).

Example 6. Jan Rokus Roosedael's "Rotations". The initial hexagon – an expression of the whole music composition

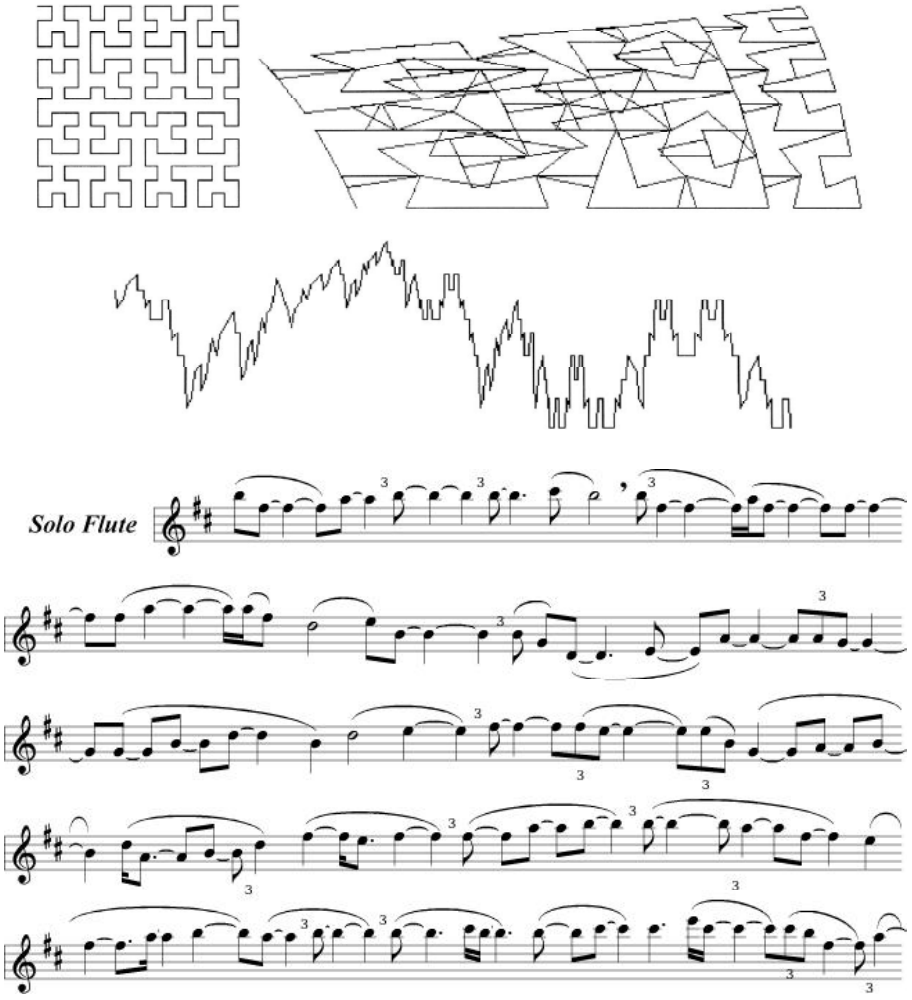


One more kind of graphical algorithm became a basis for the whole music composition. It was so called L-system curve that was the inspiration source for composer Gary Lee Nelson. The Hilbert curve was a primary model for his flute solo piece "Summer Song". The traditional curve was modified – composer changed the ordinary step of 90° (degree) to the angle of 101°. After that Nelson made a horizontal extension of the new graphic model and transferred it into the tone scale (Example 7).

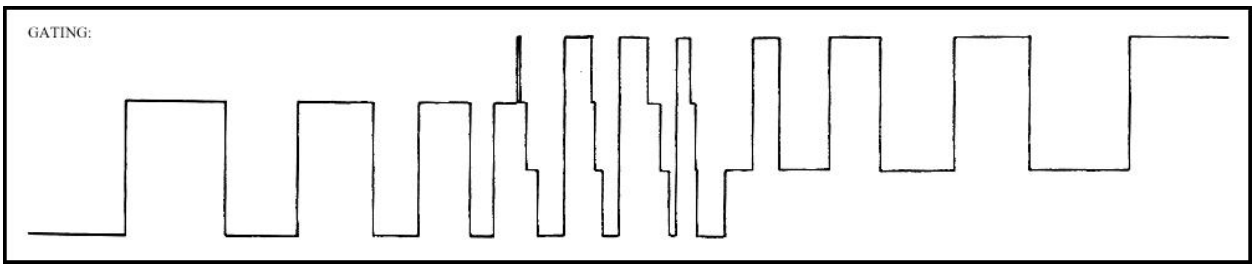
Another example is John Adams' "China Gates" score. In the beginning composer provides the symmetric curve with the remark "Gating". This composition is based on minimalism technique and the mentioned curve was used as a composing algorithm that determined the structure of the whole piece, the duration and rhythmical parameters. Every change in the curve direction means the changes of "Gating", so called the tunes after Adams (Example 8).

And for **the third aspect** I would like to provide several examples, where the mathematical model, which was adjusted to the music composition, is not only a technological thing, not only a tool for composer and determines not only the composing process. Such model becomes semantic. For example, it may be the incrustation of various meaningful geometric forms in the music score.

Example 7. Gary Lee Nelson's "Summer Song". The Hilbert curve, its modified version, linear extension and transformation into melody



Example 8. John Adam's "China Gates". The curve as an algorithm of the duration parameter



Lithuanian composer Bronius Kutavičius in various his compositions implicated the graphical models with sacral character. Composer was admired by various symbols, for example, the Celtic cross, which you may see in the composition "Kampf der Bäume" (1996) – the score of the first and the second parts is arranged in the cross form namely. The score of "Magic Circle of Sanskrit" (1990) symbolizes the circle, and the score of "Last pagan Rites" (1978) is arranged into the star-circle form. Composer George Crumb in his piece "The Star Child" used such symbols also. It is not incidental that in the two parts "Musica Mundana 1" and "Musica Mundana 2" the traditional music score is recomposed into the graphical symbol of mundi/universe – the staves are curved into the circle.

Such music notation examples show the various ways how may be used the mathematical models or elements. I may state that such phenomenon manifests from pure technological side, where it is used abstract mathematical tool, to the semantic character, where the composer uses structural mathematical model as a special semantic code for the interpretation of his composition. These various examples are unified in their background which I would define as mathematical. The question may arise – why such tendency of mathematical music may be exclusive? However, it turns to the problem of music and mathematics interaction that continues more than millennium and excites the theorists and speculators. From one side, a problem presented in this paper turns towards the renaissance conception of human as *homo ludus* (lat. the playing human). And here I would like to refer to Stravinsky's quotation: composer says better enjoys the composition of music. So this problem prompts a question: are such composing processes and in such ways composed music piece as a result devoted more to the visual/analytical purpose or to the audible/receptional purpose? Would such mathematical manipulations in music are the composers' compositional game or a game for researcher's mind only? Is it needed a mathematical explanation to the listener who perceives such music? A mathematical perceiving and cognizing of music composition may be one more way of listening to the art of sounds. But sometimes may be that composers may hide themselves under these intriguing explanations of their work and raise the interesting prototype above their result – above the music.

Santrauka

Keletas matematizuotų XX a. muzikos kompozicijos užrašymo atvejų

Pranešime pristatoma muzikos notacijos problema tęsia muzikologinius tyrinėjimus, susijusius su muzikos ir matematikos sąveika.

Muzikos ir matematikos paralelė, kilusi dar antikos laikais, kai muzika buvo priimta į *quadrivium* greta kitų trijų mokslų – astronomijos, aritmetikos ir geometrijos, įvairiomis formomis ir raiškos pobūdžiu tęsiasi per įvairias epochas; o XX a. muziką šiuo požiūriu galima apibūdinti kaip ankstesnių laikotarpių numerologinių tradicijų sintezę.

Pranešime pateikiama keletas šiuolaikinių muzikos kompozicijų pavyzdžių, kurių notacijos/užrašymo būdas interpretuojamas kaip matematizuotas. Sąvoka „matematizacija“ aprėpia įvairius konstruktyvius būdus. Tiesioginė nuoroda į muzikos kompozicijos matematizaciją būtų M. Feldmano partitūros „IXION“ notacija/užrašymas, kuris tėra tik skaičių langeliuose kombinacijos.

C. Nancarrowo studijų pianolai tempų užrašymai iliustruoja originalų kompozitoriaus mėginimą atskirus balsus tarpusavyje derinti pasitelkus sudėtingus matematinius apskaičiavimus: pvz., keturbalsės Studijos Nr. 27 pradžioje kompozitorius pateikia užrašą „Kanonas 5%-6%-8%-11%“, kuriuo nurodo, kokiu tempo santykiu jis tarpusavyje suderino keturias linijas; dvylikabalsės Studijos pianolai Nr. 37 kiekvienos linijos tempas su kitomis sutinka pagal matematinę išraišką: 150-160$^{5}/_{7}$-168¾-180-187$^{1}/_{2}$-200-210-225-240-250-262$^{1}/_{2}$-281$^{1}/_{4}$, o ši skaičių seka atitinka skaitmeninius santykius tarp garsų iš pilno (visų dvylikos garsų) chromatinio garsaeilio.

Prie muzikos kūrinio matematizuotos notacijos priskirtume grafinio algoritmo taikymą. Grafinis algoritmas interpretuojamas kaip pirmavaizdis muzikos kūrinio, vėliau užrašomo tradicine notacija. Pavyzdžiui, J. Adamso „China Gates“ partitūros pradžioje pateikiama kreivė. Kaip Š. Nako kompozicijos „Ziqquratu“ pirmavaizdžius galima nurodyti ne tik partitūroje implikuotus dviejų skaitmeninių kvadratų dėsnius, kurie lėmė atskirų instrumentų partijų ritminį piešinį, bet ir grafines figūras – remiantis kompozitoriaus pastabomis, fortepijono partijos klasterius lėmė kompozitoriaus iš anksto padaryti grafiniai brėžiniai.